**Abstracts of Invited Talks**

* **Hernan Cendra** (Universidad Nacional del Sur, Argentina), *Dirac Systems in Mechanics.*

Abstract: I will show how to use Dirac systems to deal with some examples in mechanics, like reduction and mechanics on Lie algebroids. This points in the direction that Dirac systems give a convenient unifying point of view, which includes the previous topics and many others.

* **Melvin Leok** (University of California, San Diego, USA), *Towards Discrete Distributed and Boundary Control of Interconnected and PDE Lagrangian Systems.*

Abstract: We will survey some recent progress in the construction of discrete Dirac structures and discrete Dirac mechanics, discrete Hamilton-Jacobi theory and their connections to control theory, and generating functional for Lagrangian PDE systems, and the systematic numerical analysis of Lagrangian variational integrators.

These developments lay the foundations for developing a discrete distributed and boundary control framework for interconnected and PDE Lagrangian systems. Discrete Dirac structures provide the appropriate discrete geometric structure for describing the interconnection of discrete Lagrange-Dirac mechanical systems, and by generalizing the relationship between discrete Hamilton--Jacobi theory and the Bellman equations of optimal control to the setting of interconnected Dirac mechanical systems.

Additionally, the notion of the boundary Lagrangian, which is a generating functional for Lagrangian multisymplectic PDEs, provides a natural generalization of the exact discrete Lagrangian that will play a fundamental role in the construction and analysis of variational integrators for Lagrangian PDEs. This then provides the basis for generalizing discrete Dirac mechanics for controlled interconnected systems to the setting of PDEs, and may eventually lead to systematic, geometric structure-preserving discretizations of boundary control problems.

* **David Martin de Diego** (CSIC, Madrid, Spain), *Continuous and discrete variational calculus for mechanical systems and optimal control*

Abstract: In this talk, we will see different systems arising in continuous and discrete mechanics using the same framework: generalized variational calculus. Moreover, we will consider its extension to optimal control theory.
* Olga Rossi (University of Ostrava, Czech Rep and La Trobe University, Melbourne, Australia), *Hamilton equations in jet bundles.*

Abstract: We discuss the current state of the Hamiltonian field theory in the light of old and new ideas and results.

* Willy Sarlet (Ghent University, Belgium and La Trobe University, Melbourne, Australia), *The search for differential ideals in the inverse problem of the calculus of variations.*

Abstract: An exterior differential system (EDS) approach is one of many techniques which has been promoted in the context of the inverse problem of the calculus of variations. For the case of second-order, ordinary differential equations, the inverse problem can be seen as the construction of a closed 2-form within the module of so-called Kähler lifts. In applying EDS to this class of problems, one can distinguish three stages. We focus on the first stage which is the construction of a submodule of the module of Kähler lifts, generating a differential ideal. The hope is that such an approach may be fruitful in coming to a classification of favourable and unfavourable situations, something which so far has only been achieved by Douglas for the 2-dimensional case. One of the difficulties in carrying out an algorithmic process to construct a differential ideal is the general question of identifying necessary and sufficient conditions for a 3-form to belong to the ideal generated by a given module of 2-forms. We explain how far we can get in fairly general terms for this process and briefly discuss further work in progress at La Trobe University.

* Hiroaki Yoshimura (Waseda University, Japan), *Dirac Structures in Vakonomic Mechanics*

Abstract: We explore vakonomic mechanics on both Lagrangian and Hamiltonian sides in the context of the Dirac structures as well as variational principles. To do this, we begin with a regular Lagrangian and with nonholonomic constraints. Then, we develop a variational principle for an extended Lagrangian on the extended tangent bundle. In particular, we construct a vakonomic Dirac structure on the extended tangent bundle by introducing a vakonomic Lagrangian two-form, which naturally induces intrinsic Lagrange-Dirac vakonomic systems. Furthermore, we show how the constrained Dirac structure can be developed by restricting the vakonomic Dirac structure to the extended constraint subspace and also show the vakonomic constrained Dirac-Lagrange system. For the case of linear constraints, choosing an Ehresmann connection, we will illustrate the local Lagrange-Dirac vakonomic system and we will finally clarify the link between Dirac structures and Lagrangian submanifolds in vakonomic mechanics. We will also show a similar procedure on the Hamiltonian side for the vakonomic Hamilton-Dirac systems. This work is a joint work with Fernando Jimenez.
Abstracts of Contributed Talks

* Hassan Alishah (Instituto Superior Tecnico, Lisbon, Portugal), *Lie algebroids and KAM theorem.*

Abstract: I will present a new approach to the KAM problem for a general vector field via Lie algebroids. I will explain how the problem of persistence of invariant tori can be restated as a problem of stability of leaves of Lie algebroids. Then, I will state a conjecture concerning the stability of compact invariant submanifolds of a Lie algebroid. I will provide some examples supporting this conjecture and will discuss possible approaches to prove this conjecture.

* Ioan Bucataru (Alexandru Ioan Cuza University, Romania), *Sprays metrizable by Finsler functions of constant curvature.*

Abstract: In this paper we solve the following particular case of the Finsler metrizability problem. Given a homogenous system of second order ordinary differential equations, we provide the necessary and sufficient conditions that can be used to decide whether or not the system represents the Euler-Lagrange equations of a Finsler function of non-zero constant curvature. The conditions we provide are tensorial equations that have to be satisfied by the Jacobi endomorphism. We identify the class of homogenous SODE where Finsler metrizability is equivalent with the metrizability by a Finsler function of non-zero constant flag curvature.

* Manuel de Leon (CSIC, Madrid, Spain), *The Hamilton-Jacobi equation for Jacobi dynamical systems.*

* Francesco Fasso (University of Padua, Italy), *Relative invariant tori.*

Abstract: We discuss reconstruction from quasi-periodic reduced dynamics. A criterion that ensures that the reconstructed dynamics is quasi-periodic is given and the structure of the `relative' invariant tori is discussed, with attention to the dimension of the resulting `minimal' invariant tori. Joint work with A. Giacobbe and L. Garcia-Naranjo.

* Francois Gay-Balmaz (Ecole Normale Superieure, Paris, France), *Lagrange-d'Alembert principle for the dynamics of elastic rods in perfect friction contact.*

Abstract: We derive the equations of motion for rolling elastic strings in persistent rolling contact. The equations, presented first in an abstract form, are obtained by using the theory of Euler-Poincare reduction by symmetries, appropriately modified to incorporate nonholonomic rolling conditions via the Lagrange-d'Alembert principle. We then show how to apply that theory to a particular case of elastic rods in rolling contact with naturally circular cross-section, when the deformation of the cross-section at contact is assumed to
be negligible. We also derive a consistent geometric theory of rolling motion for discrete rods, or chains, in contact.

* **Eduardo García-Torano** (Ghent University, Belgium), *Some transformations in the framework of Routh reduction.*

Abstract: In this talk we will introduce some transformations arising in the context of Routh reduction. We will show that under suitable conditions, they relate the dynamics of the reduced systems and can therefore be used to state the equivalence of reduction by different symmetry groups.

* **Demeter Krupka** (University of Ostrava and Lepage Research Institute, Czech Rep), *Lepage forms in Kawaguchi spaces and the Hilbert form.*

Abstract: A well-known construction in geometric mechanics and Riemann-Finsler geometry assigns to a (first order) homogeneous Lagrangian the Hilbert form, serving as an integrand in the corresponding variational functional. Analogous constructions, needed for higher-order mechanics and Finsler-Kawaguchi geometry, have not been found yet. In this paper we construct Lepage equivalents of Lagrangians, satisfying higher-order homogeneity (Zermelo) condition. We show that the homogeneity determines uniquely higher-order momenta and annihilates local Hamiltonians. The resulting Lepage equivalents then represent higher-order generalizations of the Hilbert form. This result extends geometric foundations of variational theory to higher-order parameter-invariant variational functionals.

* **David Saunders** (University of Ostrava, Czech Rep), *Some recent developments in the theory of parametric variational problems.*

Abstract: Both classical mechanics and Finsler geometry give rise to problems in the calculus of variations. They are, however, problems of different types: in mechanics, the extremals are curves with a given parameter (time), whereas in Finsler geometry the extremals are geometric curves without any given parametrization, although perhaps with an orientation. The latter type of problem is called a parametric variational problem, and may also be defined with several independent variables and with dependence on higher order derivatives.

In Finsler geometry, the variational problem may be described in two different ways, as a problem on the projective tangent bundle or the sphere bundle, and alternatively as a problem on the slit tangent bundle. The same distinction holds for more general parametric variational problems, and in this talk I shall describe some developments in both the theory of the manifolds involved, and in the application to the calculus of variations.
Joris Vankerschaver (Imperial College, UK), An Euler-Poincare description of curve matching.

Abstract: In image analysis, the following question often arises: given two curves in the plane (or two images, or other objects), how (dis)similar are these curves? One way to answer that question is to select a Lie group of transformations, and to deform the first curve into the second by means of (parameter-dependent transformations). Given a suitable deformation energy on the Lie group, a measure for dissimilarity or discrepancy is then given by the infimum of the energy needed to deform one curve into the other.

I will discuss the case of two curves in the plane, for which the Lie group is SE(2), the group of Euclidian isometries. I will show that the energy minimizer satisfies a set of constrained Euler-Poincare equations on the Lie algebra of SE(2), and that this system can be viewed as a Chaplygin system with (time-dependent) holonomic constraints. I will illustrate this theory with a number of examples. This is joint work with Lyle Noakes and Darryl Holm.

Sergiy Vasylkevych (Jacobs University Bremen, Germany), Lagrangian modified equations for backward error analysis of variational integrators.

Abstract: The construction of modified equations is an important step in the backward error analysis of symplectic integrators for Hamiltonian systems. Usually modified equations are constructed on the Hamiltonian side. In the context of PDEs, the standard construction leads to equations supporting increasingly higher frequency modes, so that strong regularity assumptions are necessary for their analytical treatment. We propose a new method of constructing modified equation, based on carefully chosen configuration space transformation. As a proof-of-concept we construct modified equations for the implicit midpoint rule applied to the semilinear wave equation working directly with the variational principle. Contrary to the standard Hamiltonian approach, this yields a modified system inheriting its analytical properties from the original wave equation.

Goedele Waeyaert (Ghent University, Belgium), Driven cofactor systems and Hamilton-Jacobi separability.

Abstract: This talk is based on joint work with W. Sarlet [1]. We start by introducing the notion of a special conformal Killing tensor. These tensors play an important role in the intrinsic characterization of so-called cofactor systems, which are special classes of nonconservative systems [3]. Next, we will describe driven cofactor systems, which are partially decoupling second-order differential equations of a special kind, and give a summary of the main results in [2]. Finally, we explain how one can reduce the Hamilton-Jacobi problem of the (a priori time-dependent) driven part of the system into that of an equivalent autonomous system of Stackel type.
References:
